A Logic for Graphs with QoS

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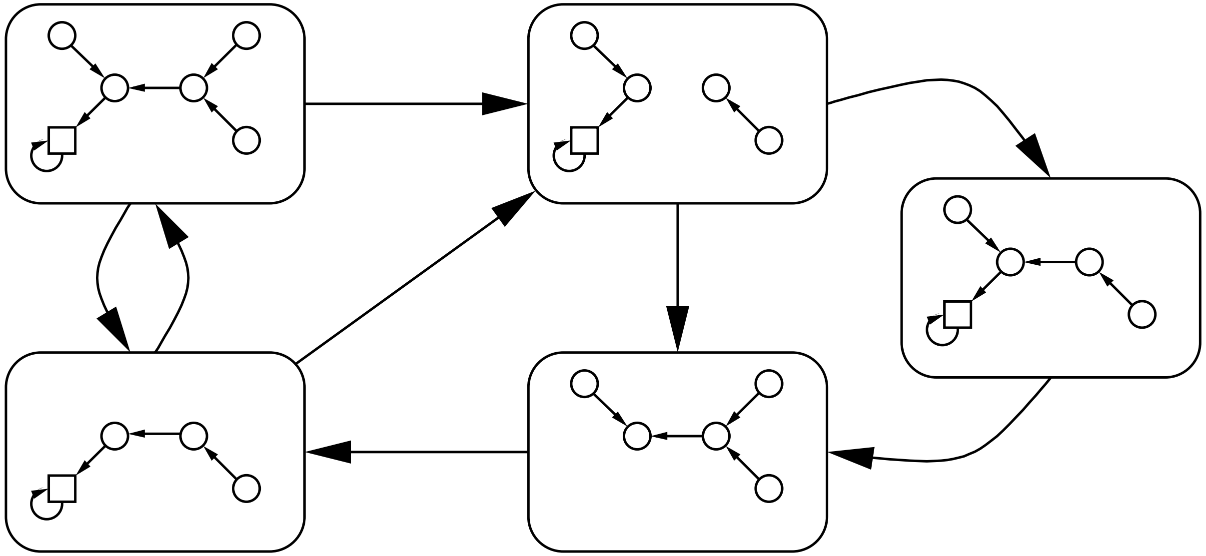
Introduction

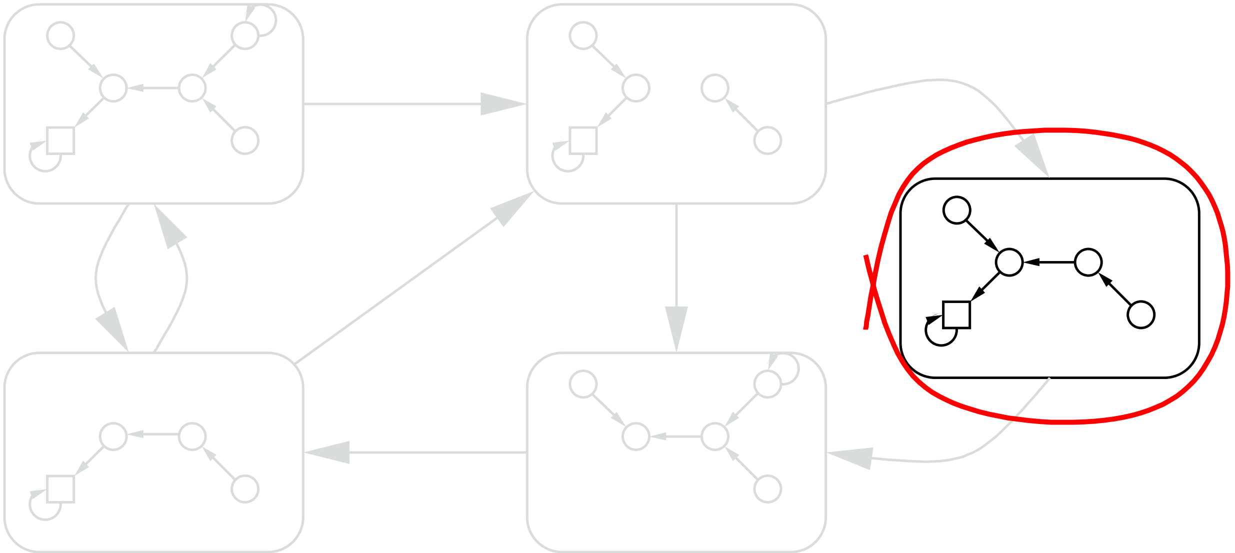
- We are interested in Specification and Verification of WAN applications.
- Systems involving concurrency, distribution, mobility and . . . QoS.
- QoS: non-functional aspects, like network bandwidth, prize, etc.
- Service Overlay Network Applications are an example.
- Graph Transformation Systems are suitable modeling formalisms.

Graph Transition System



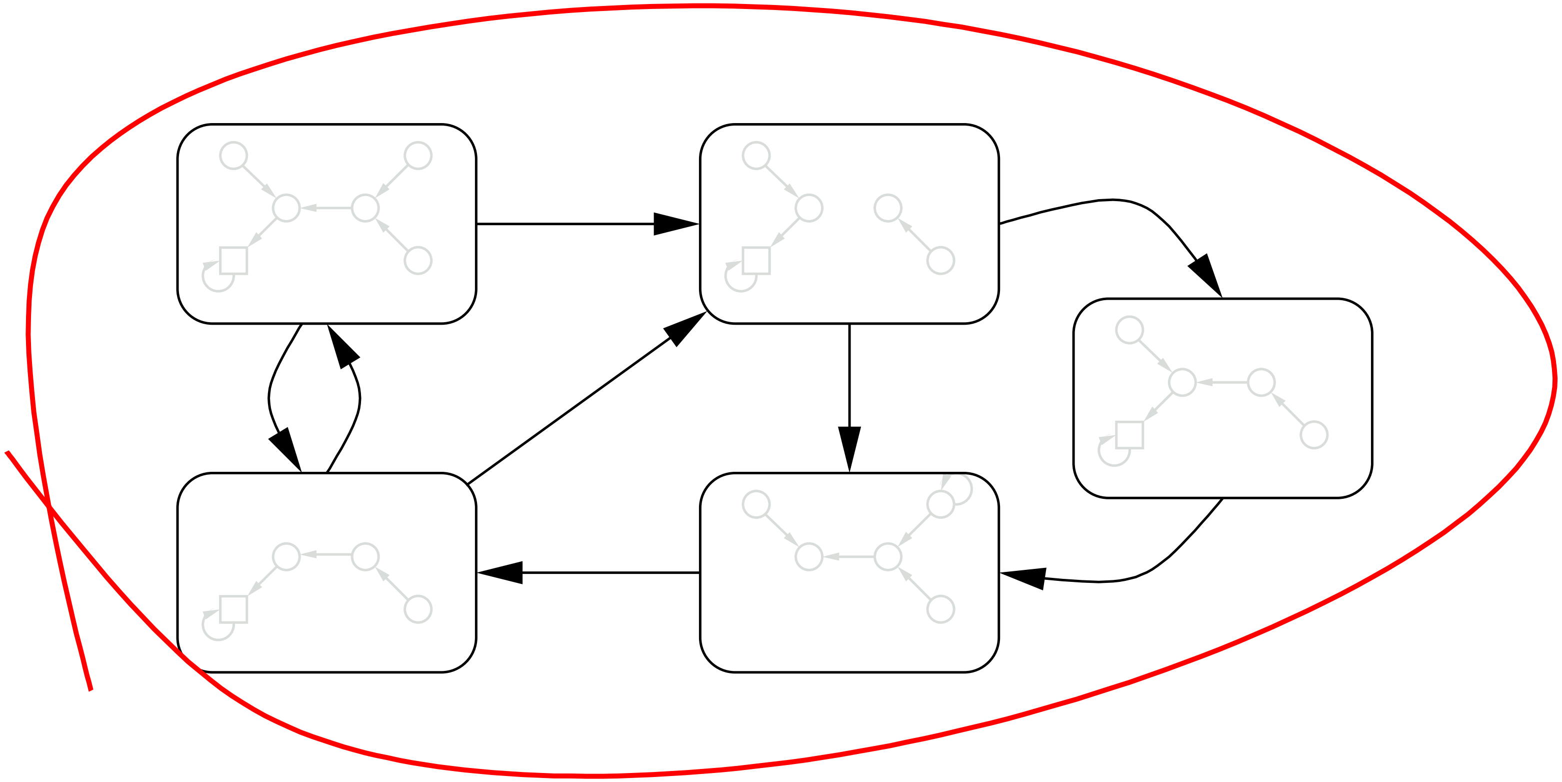
- Transition system where each state is (or has a associated) a graph representing, for instance, the interconnection between components.

Graph Logics



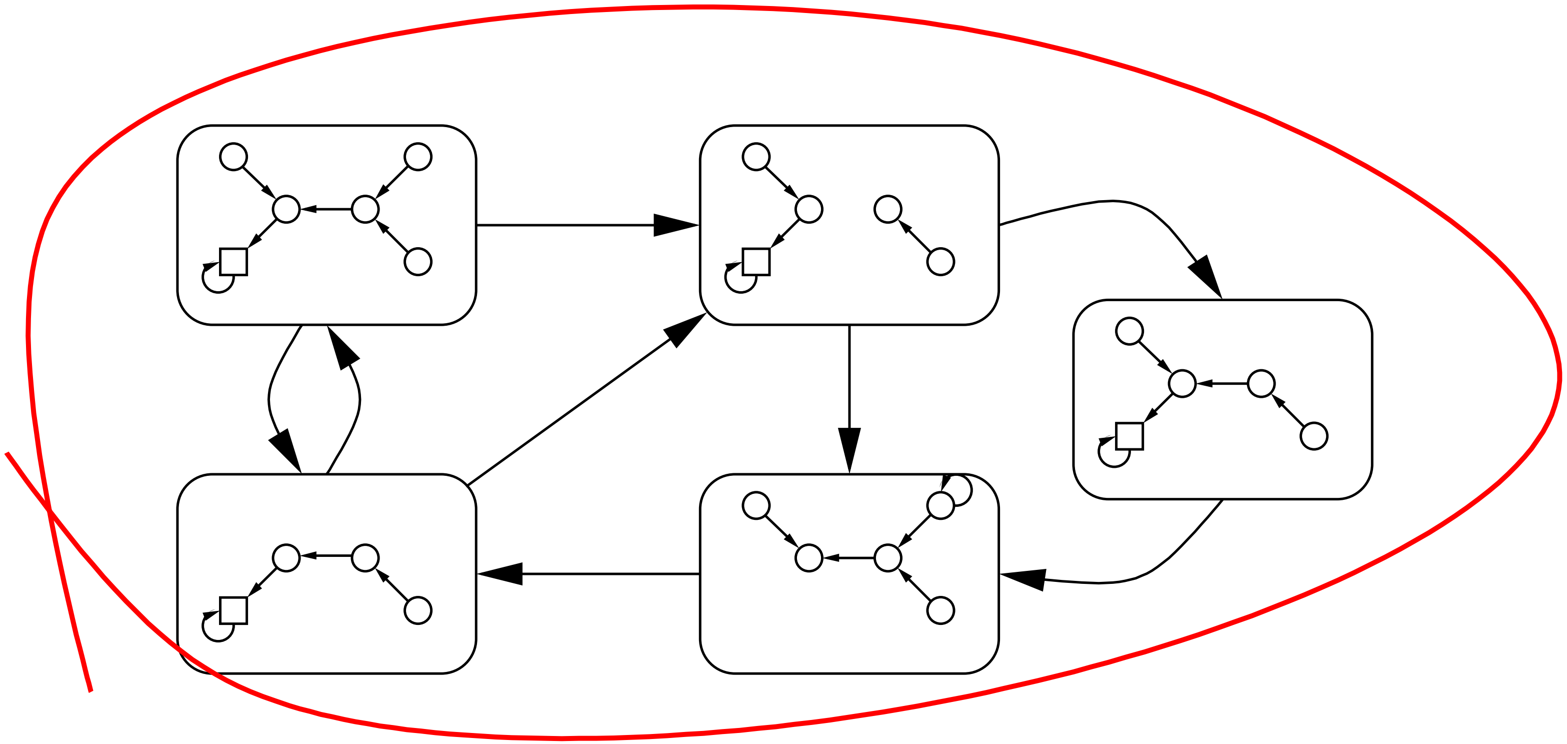
- Graph Logics allow to express properties of graphs.
- Examples: MSO (and its fragments), GL.

Temporal Logics



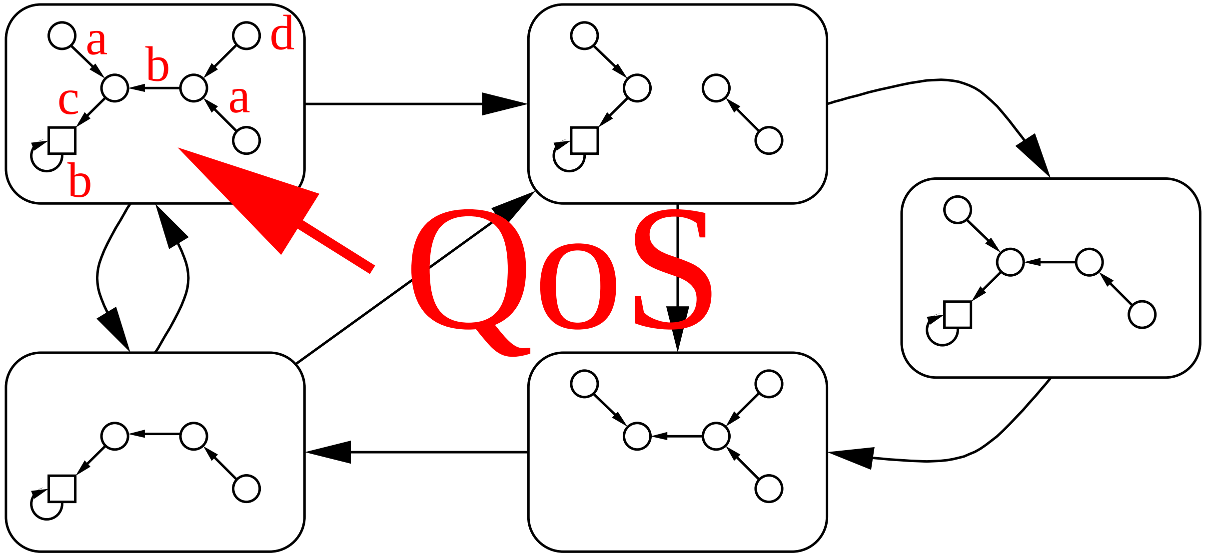
- Temporal logics allow to reason about the ordering of events in time.
- Examples: μ-calculus, CTL*, CTL, LTL.

Logics for GTSs



- Allow to reason about the evolution of graphs in time.
- Examples: Courcelle, Baldan et. al, A. Rensink, etc.

Systems with QoS



- Graphs enriched with QoS attributes.
- We need logics to reason about QoS.

Our formal model for QoS: c-semirings

- Each attribute represented by a (partially) ordered set of values.
- An operation to select among values (generalized addition, or min).
- An operation to combine values (generalized multiplication).
- Support of composition of different attributes (multicriteria).

C-semirirings: Definition

A c-semiring is a tuple $\langle A, +, \times, \mathbf{0}, \mathbf{1} \rangle$ such that:

- A is a set;
- $\mathbf{0}$ and $\mathbf{1}$ are elements of A;
- $+ : 2^A \rightarrow A$ is defined over (possibly infinite) sets of elements of A as follows[a]: $\sum\{a\} = a$, $\sum \emptyset = \mathbf{0}$, $\sum A = \mathbf{1}$ and $\sum(\bigcup A_i) = \sum\{\sum A_i\}$, for $A_i \subseteq A$, $i \geq 0$;
- $\times : A \times A \rightarrow A$ is a binary associative, commutative operation that distributes over $+$, has $\mathbf{1}$ as unit element and $\mathbf{0}$ as absorbing element.

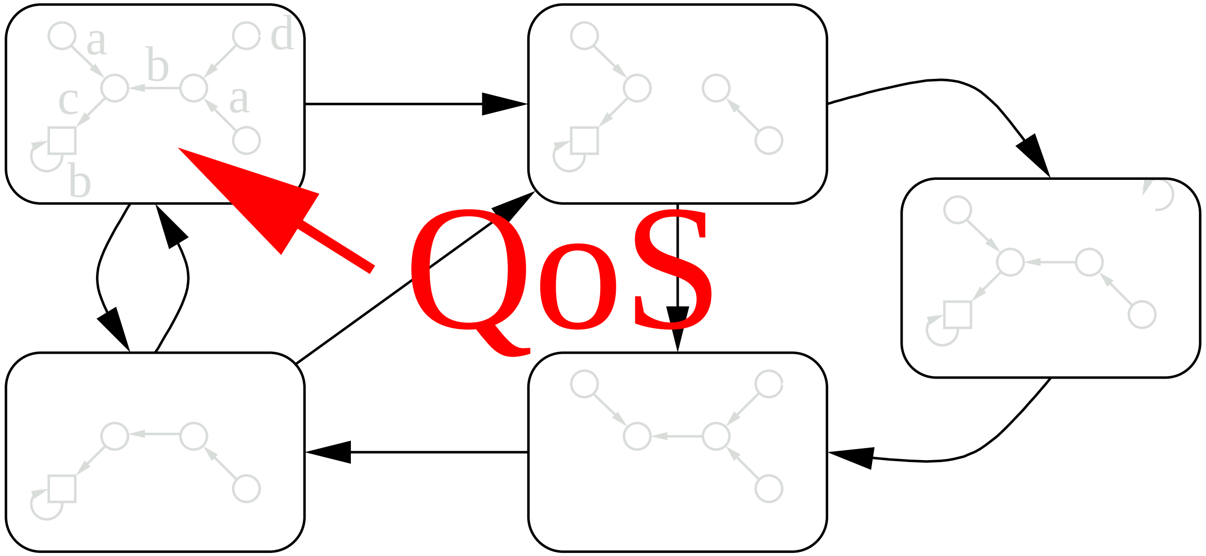
[a] When $+$ is applied to a set with two elements we use $+$ as binary operator in infix notation, while in all other cases we use symbol $\sum$ in prefix notation.

C-semirings: Examples

C-semirings are the formal structure of many QoS attributes.

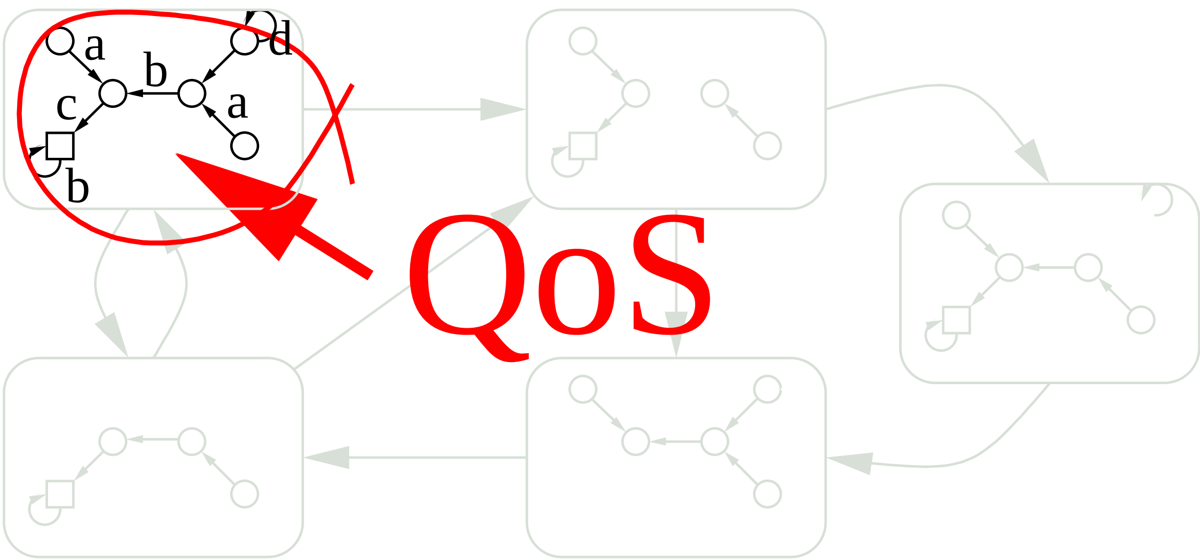
A	$+$	$\times$	$\mathbf{0}$	$\mathbf{1}$	Example
$\{T, F\}$	$\vee$	$\wedge$	F	T	Network and service availability
$\mathbb{R}^+$	min	$+$	$+\infty$	0	Price, propagation delay
$\mathbb{N}^+$	min	$+$	$+\infty$	0	Price, propagation delay
$\mathbb{R}^+$	max	min	0	$+\infty$	Bandwidth
$[0, 1]$	max	$\cdot$	0	1	Performance
$[0, 1]$	max	min	0	1	Performance, security degree
2^N	$\cup$	$\cap$	$\emptyset$	N	Capabilities, access rights

Temporal Logics for QoS



- A formula is evaluated as a c-semiring value.
- Basically, $\vee$ and $\wedge$ are substituted by $+$ and $\times$.
- Eventually x and y connected
  $\Rightarrow$ Optimal QoS of connection between x and y ever achieved.

Graph Logic for QoS



Again...

- A formula is evaluated as a c-semiring value.
- Basically, $\vee$ and $\wedge$ are substituted by $+$ and $\times$.
- x are y connected
  $\Rightarrow$ Best QoS of connection between x and y.

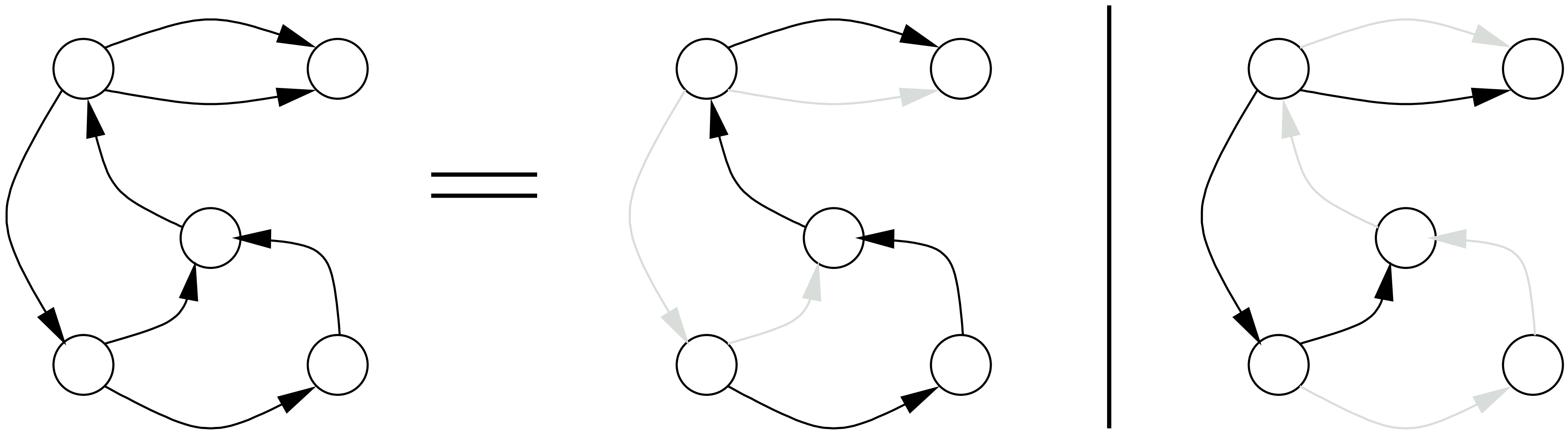
Graph Model: Definition

A graph is a tuple $\langle X \cup E, \mathit{edge}, C, K, I \rangle$, where:

- $X \subseteq \mathcal{X}$ is a (finite) set of nodes,
- $E \subseteq \mathcal{E}$ is a (finite) set of edges,
- $\mathit{edge} : E \to X \times X$ associates edges with source and target,
- C is a c-semiring,
- $K \subseteq \mathcal{K}$ is a set of cost function symbols,
- I is an interpretation of symbols.

Graph Model: Decomposition

A decomposition of G is a pair of graphs G_1, G_2 such that $E_1 \uplus E_2 = E$, $edge_1 \uplus edge_2 = edge$.



The set of all decompositions of a graph G will be denoted by $\Theta(G)$.
The size of $\Theta(G)$ is $2^{|E|}$.

Graph Logic: Syntax

Let V_X be a set of node variables, and V_R be a set of recursion variables.

ϕ	$::=$	$\mathbf{nil} \mid k(\xi) \mid k(\xi,\xi) \mid \phi|\phi \mid \phi\|\phi$	spatial operators
		$\phi{+}\phi \mid \phi\times\phi \mid f(\phi,\ldots,\phi)$	c-semiring operators
		$\sum \mathbf{x}.\phi \mid \prod \mathbf{x}.\phi$	quantification
		$\mathbf{r}(\overline{\xi}) \mid (\mu\mathbf{r}(\overline{\mathbf{x}}).\phi)\overline{\xi} \mid (\nu\mathbf{r}(\overline{\mathbf{x}}).\phi)\overline{\xi}$	fixpoints

where $x \in \mathcal{X}$, $\mathbf{x} \in V_X$, $\xi \in \mathcal{X} \cup V_X$, $k \in \mathcal{K}$, $f \in \mathcal{F}$, $\mathbf{r} \in V_R$.

Graph Logic: Semantics (void and costs)

$$[\![\mathbf{nil}]\!]_{\sigma;\rho}(G) \;=\; (E = \emptyset)$$

1 if the graph has no edges, **0** otherwise.

$$[\![k(\xi)]\!]_{\sigma;\rho}(G) \;=\; I(k)(x\sigma)$$

k-cost of node

$$[\![k(\xi_1, \xi_2)]\!]_{\sigma;\rho}(G) \;=\; (E = \{e\}) \times (\mathit{edge}(e) = (\xi_1\sigma, \xi_2\sigma)) \times I(k)(e)$$

k-cost of edge (if it is the only one in the graph, else **0**).

Graph Logic: Semantics (decomposition)

$$[\![\phi_1|\phi_2]\!]_{\sigma;\rho}(G) \;=\; \sum_{(G_1,G_2)\in\Theta(G)} \{[\![\phi_1]\!]_{\sigma;\rho}(G_1) \times [\![\phi_2]\!]_{\sigma;\rho}(G_2)\}$$

$$[\![\phi_1\|\phi_2]\!]_{\sigma;\rho}(G) \;=\; \prod_{(G_1,G_2)\in\Theta(G)} \{[\![\phi_1]\!]_{\sigma;\rho}(G_1)+[\![\phi_2]\!]_{\sigma;\rho}(G_2)\}$$

Graph Logic: Semantics (c-semiring operations)

$$[\![\phi_1 + \phi_2]\!]_{\sigma;\rho}(G) \;=\; [\![\phi_1]\!]_{\sigma;\rho}(G) + [\![\phi_2]\!]_{\sigma;\rho}(G)$$

$$[\![\phi_1 \times \phi_2]\!]_{\sigma;\rho}(G) \;=\; [\![\phi_1]\!]_{\sigma;\rho}(G) \times [\![\phi_2]\!]_{\sigma;\rho}(G)$$

$$[\![f(\phi_1, \ldots, \phi_n)]\!]_{\sigma;\rho}(G) \;=\; I(f)([\![\phi_1]\!]_{\sigma;\rho}(G), \ldots, [\![\phi_n]\!]_{\sigma;\rho}(G))$$

Graph Logic: Semantics (Quantification)

$$[\![\sum \mathbf{x}.\phi]\!]_{\sigma;\rho}(G) \;=\; \sum_{\mathbf{x}\in X} [\![\phi]\!]_{\sigma;\rho}(G)$$

$$[\![\prod \mathbf{x}.\phi]\!]_{\sigma;\rho}(G) \;=\; \prod_{\mathbf{x}\in X} [\![\phi]\!]_{\sigma;\rho}(G)$$

Graph Logic: Semantics (fixpoints)

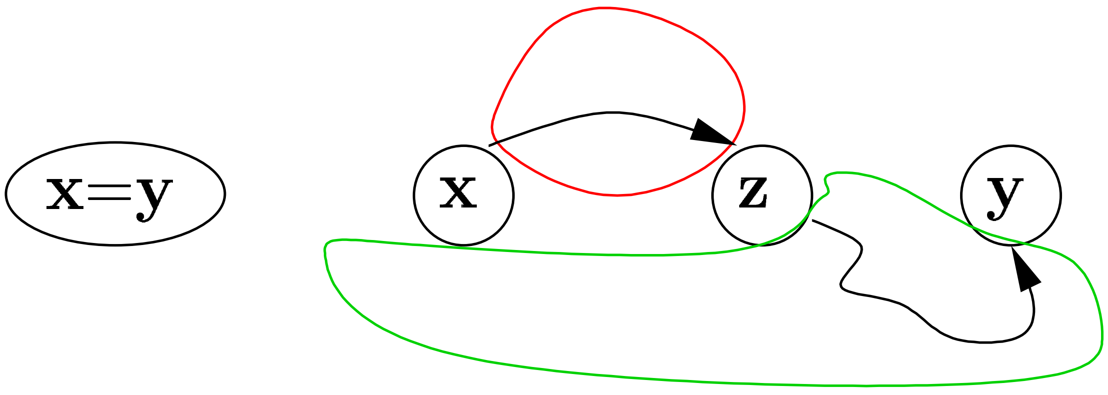
$$[\![\mathbf{r}(\overline{\xi})]\!]_{\sigma;\rho}(G) \;=\; \mathbf{r}\rho(\overline{\xi}\sigma)$$

$$[\![(\mu\mathbf{r}(\overline{\mathbf{x}}).\phi)\overline{\xi}]\!]_{\sigma;\rho}(G) \;=\; \mathit{lfp}(\lambda\mathbf{s}.\lambda\overline{\mathbf{y}}.[\![\phi]\!]_{\sigma[\overline{\mathbf{y}}/\overline{\mathbf{x}}],\rho[\mathbf{s}/\mathbf{r}]})(\overline{\xi}\sigma)(G)$$

$$[\![(\nu\mathbf{r}(\overline{\mathbf{x}}).\phi)\overline{\xi}]\!]_{\sigma;\rho}(G) \;=\; \mathit{gfp}(\lambda\mathbf{s}.\lambda\overline{\mathbf{y}}.[\![\phi]\!]_{\sigma[\overline{\mathbf{y}}/\overline{\mathbf{x}}],\rho[\mathbf{s}/\mathbf{r}]})(\overline{\xi}\sigma)(G)$$

There is a path from x to y

$$\mathbf{path}(\mathbf{x},\mathbf{y}) \equiv \mu\mathbf{r}(\mathbf{x},\mathbf{y}).(\mathbf{x}=\mathbf{y}) + \sum \mathbf{z}.e(\mathbf{x},\mathbf{z})|\mathbf{r}(\mathbf{z},\mathbf{y}).$$



QoS of optimal path from x to y

$$\mathbf{costp}(\mathbf{x},\mathbf{y}) \equiv \mu\mathbf{r}(\mathbf{x},\mathbf{y}).(\mathbf{x}=\mathbf{y}) + \sum \mathbf{z}.\mathit{cost}(\mathbf{x},\mathbf{z}))|\mathbf{r}(\mathbf{z},\mathbf{y})$$

Graph Logic: Verification

- GL is PSPACE-complete (with and without recursion).
- Without recursion (abstracting from c-semiring operations):
  - **nil**, $k(\xi)$ and $k(\xi, \xi)$ can be done in $O(1)$.
  - $\sum \mathbf{x}.\phi$ and $\prod \mathbf{x}.\phi$ require $O(|X|)$.
  - $\phi|\phi$ and $\phi||\phi$ require to consider all the $2^{|E|}$ decompositions.
    $\Rightarrow$ this leads to $O(2^{|E|^{|\phi|}})$.
- With recursion:
  - Iteration might not terminate.
  - Combine recursion with composition.
    $\Rightarrow$ Conjecture: Guarantees termination.
    $\Rightarrow$ Open question: Limits expressivity of *practical* problems?

Conclusion

- We have extended a graph logic to include QoS attributes.
- We have temporal logics to reason about QoS in time.
- We can define a logic to reason about spatial, temporal and QoS properties of systems.